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NOTE

Evaluation of CSA S16-14 asymmetry parameter for singly-symmetric beams

James C. Koch, Robert G. Driver, Yong Li, and Michael Manarin

Abstract: The provisions in the Canadian design standard S16, *Design of Steel Structures*, for determining the lateral–torsional buckling capacity of unequal-flange I-shaped beams employ an asymmetry parameter, β_x, that is a function of the cross-sectional shape. It has been observed that the approximate equation for this parameter in the standard can be highly inaccurate in some cases. A study was completed to compare the approximate values of β_x to the exact solutions for 16 312 singly-symmetric I-sections and 188 standard WT-shapes from the CISC *Handbook of Steel Construction*. It is concluded that the current standard provides extremely variable results, but is generally conservative for use in design, with the exception of T-sections, as long as the larger flange is in compression. Recommendations for adoption into the next edition of the standard are provided.

Key words: asymmetry parameter, lateral–torsional buckling, singly-symmetric, WT-shapes.

Résumé : Les dispositions de la norme de conception canadienne S16, conception de structures d'acier, pour déterminer la résistance au flambage latéral torsionnel d'une poutre en I de membrures inégales emploient un paramètre d'asymétrie, β_x, qui est une fonction de la forme de section transversale. Il fut constaté que l'équation approximative pour ce paramètre selon la norme peut être très inexacte dans certains cas. Une étude fut réalisée afin de comparer les valeurs approximatives de β_x avec les solutions exactes pour 16 312 sections en I monosymétriques et 188 formes WT standard du « CISC *Handbook of Steel Construction* ». On en conclut que la norme actuelle fournit des résultats extrêmement variables, mais qu'elle est généralement prudente au niveau de son utilisation dans la conception, à l'exception de sections en T, tant que la plus grande membrure est en compression. Les recommandations en vue de leur adoption dans la prochaine édition de la norme sont fournies. [Traduit par la Rédaction]

Mots-clés : paramètre d'asymétrie, flambage latéral torsionnel, monosymétrique, formes WT.

Introduction

Lateral–torsional buckling (LTB) can be the governing ultimate limit state for I-shaped beams in a wide variety of typical structures. This phenomenon must therefore be understood by the structural engineer to ensure safe and efficient designs. The behaviour of doubly-symmetric I-sections susceptible to global instability is well-researched, and provisions in modern design standards (e.g., CSA 2014*a*; AISC 2016) are fairly consistent and easy to apply. However, LTB response of beams with singly-symmetric I-sections tends to be significantly more complex, requiring additional theoretical knowledge to ensure that design provisions are appropriately applied.

An early theoretical overview of the challenges of evaluating the stability of singly-symmetric I-shaped beams was introduced by Anderson and Trahair (1972), who also presented experimental results on elastic LTB of singly-symmetric cantilever beams. The main difference between the LTB behaviour of beams with doubly-symmetric and singly-symmetric I- and T-sections arises due to the so-called "Wagner effect". This effect—identified initially by Wagner (1936) in the context of torsional buckling of columns—is produced by the twisting of a member, which causes axial stresses to exert either a disturbing or restraining torque depending on whether they are, respectively, compressive or tensile. This torque can therefore be considered to decrease or increase the torsional stiffness of the member depending on the sign of the axial stress. For LTB of beams with doubly-symmetric I-sections, the symmetric nature of the axial compressive and tensile stresses render the Wagner effect inconsequential (Anderson and Trahair 1972). Conversely, beams with singly-symmetric I-sections experience either a decrease or an increase in torsional stiffness depending on which flange is in compression.

The complex nature of LTB of singly-symmetric I-sections has led to proposals for simplifying the calculations of certain required cross-sectional parameters for design applications. In particular, the Canadian design standard (CSA 2014*a*) provides an approximate equation for the asymmetry parameter, β_x, which is a geometric variable required to solve the governing elastic differential equations for LTB of beams. It has been noted that for extreme singly-symmetric I-sections, this approximation can result in large errors in the value of β_x and this paper examines the extent and nature of these errors by comparing the current provisions of the standard (CSA 2014*a*) with the closed-form solution.

Background and literature review

A comprehensive review of LTB can be found in structural engineering textbooks (e.g., Galambos 1968), and only a brief overview of the most relevant concepts is included here. For an initially-straight prismatic singly-symmetric beam with end moments acting about the major principal cross-sectional axis, elas-

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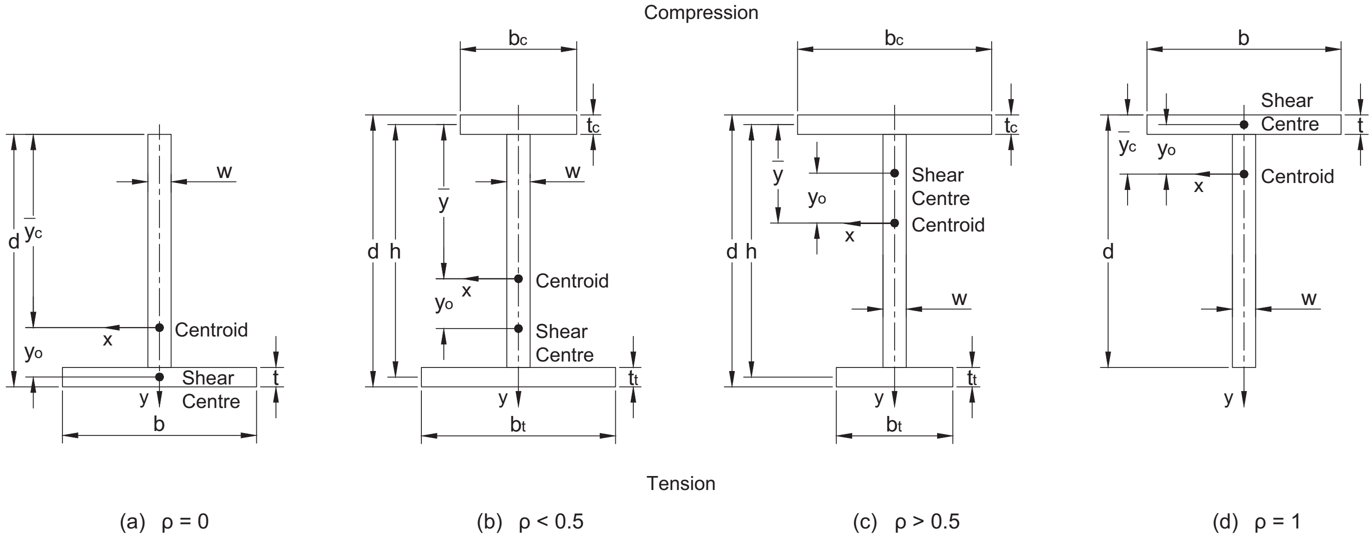
Fig. 1. Singly-symmetric I-sections and symbol definitions: (*a*), (*b*) correspond to negative β_x and (*c*), (*d*) correspond to positive β_x.



tic LTB can be described by the following coupled governing differential equations, where the principal cross-sectional axes are defined as the *x*- (major) and *y*- (minor) axes and the origin is positioned at the geometric centroid of the cross-section:

$$EI_y u^{\mathrm{iv}} + M_x \alpha'' + 2M'_x \alpha' = 0 \tag{1}$$

$$EC_{\mathrm{w}} \alpha^{\mathrm{iv}} - (GJ + M_x \beta_x)\alpha'' - M'_x \beta_x \alpha' + M_x u'' = 0 \tag{2}$$

where E is the modulus of elasticity, I_y is the cross-sectional second moment of area about the *y*-axis, u is the lateral deflection of the beam shear centre, M_x is the internal bending moment about the *x*-axis, α is the cross-sectional rotation about the longitudinal beam axis (twist), C_{w} is the warping torsional constant, G is the elastic shear modulus, J is the St. Venant torsional constant, and the prime symbol indicates differentiation with respect to the longitudinal axis. The asymmetry parameter referenced to the axis about which bending takes place, β_x, is defined as[1]

$$\beta_x = \frac{1}{I_x}\int_A y(x^2 + y^2)\mathrm{d}A - 2y_0 \tag{3}$$

where I_x is the cross-sectional second moment of area about the *x*-axis, A is the cross-sectional area, x and y represent the coordinates along the *x*- and *y*-axes of any point on the cross-section, and y_0 is the *y*-coordinate of the cross-sectional shear centre with respect to the centroid. Relevant cross-sectional dimensions are defined in Fig. 1, where a variety of singly-symmetric I-sections are depicted, including the extreme case of a T-section. (It is noted that *y*-dimensions in Fig. 1 that are topped with a bar symbol are to be treated in the following discussion as absolute values.) The *y*-axis is taken as the axis of symmetry with a positive orientation downward, and the moment is assumed to be applied about the *x*-axis with the resulting compression and tension zones identified in Fig. 1 for the special case of positive bending. The parameter $\rho = \frac{I_{yc}}{I_{yc}+I_{yt}}$, where I_{yc} and I_{yt} are the *y*-axis second moments of area of the compression and tension flanges, respectively, is commonly used as a convenient way to characterize the relative sizes of the compression and tension flanges, varying from 0 (T-section with the flange in tension) to 0.5 (doubly-symmetric section) to 1.0 (T-section with the flange in compression). Although β_x is nominally a cross-sectional parameter, in the form presented here it also possesses a sign that effectively identifies whether compression occurs in the larger (positive β_x) or smaller (negative β_x) flange, as indicated for positive bending in Fig. 1. This sign convention is equally valid for negative bending, as well as when the moment reverses sign within the unbraced beam segment. In the latter case, the signs of the bending moment and β_x reverse at the same point along the beam.

A fundamental difference between doubly- and singly-symmetric I-sections is that for the former the shear centre and centroid coincide, resulting in a value of zero for both y_0 and, as can be deduced from eq. (3), β_x. This outcome allows a significant simplification in the governing differential equations that leads ultimately to simpler and more transparent provisions in design standards. (A similar outcome, where $\beta_x = 0$, occurs for singly-symmetric sections bent about their axis of symmetry.) For singly-symmetric I- and T-sections bent about the *x*-axis, β_x can be expanded based on the geometry of these sections, with fillets neglected, as

$$\beta_x^I = \frac{1}{I_x}\left[(h - \bar{y})\left(\frac{b_t^3 t_t}{12} + b_t t_t (h - \bar{y})^2 + \frac{w}{4}(h - \bar{y})^3\right) - \bar{y}\left(\frac{b_c^3 t_c}{12} + b_c t_c \bar{y}^2 + \frac{w}{4}\bar{y}^3\right)\right] - 2y_0 \tag{4}$$

$$\beta_x^T = \frac{1}{I_x}\left[\frac{w}{4}\left((d - \bar{y}_c)^4 - \left(\bar{y}_c - \frac{t}{2}\right)^4\right) - bt\left(\bar{y}_c - \frac{t}{2}\right)\left(\frac{b^2}{12} + \left(\bar{y}_c - \frac{t}{2}\right)^2\right)\right] - 2y_0 \tag{5}$$

where the superscript applied to β_x is indicative of the cross-sectional shape.

The asymmetry parameter, β_x, in the current Canadian design standard (CSA 2014*a*) for singly-symmetric I-sections is based on the work of Kitipornchai and Trahair (1980), who proposed the following form (assuming the flanges have the same thickness, t):

[1]Supplementary data are available with the article through the journal Web site at http://nrcresearchpress.com/doi/suppl/10.1139/cjce-2018-0223.

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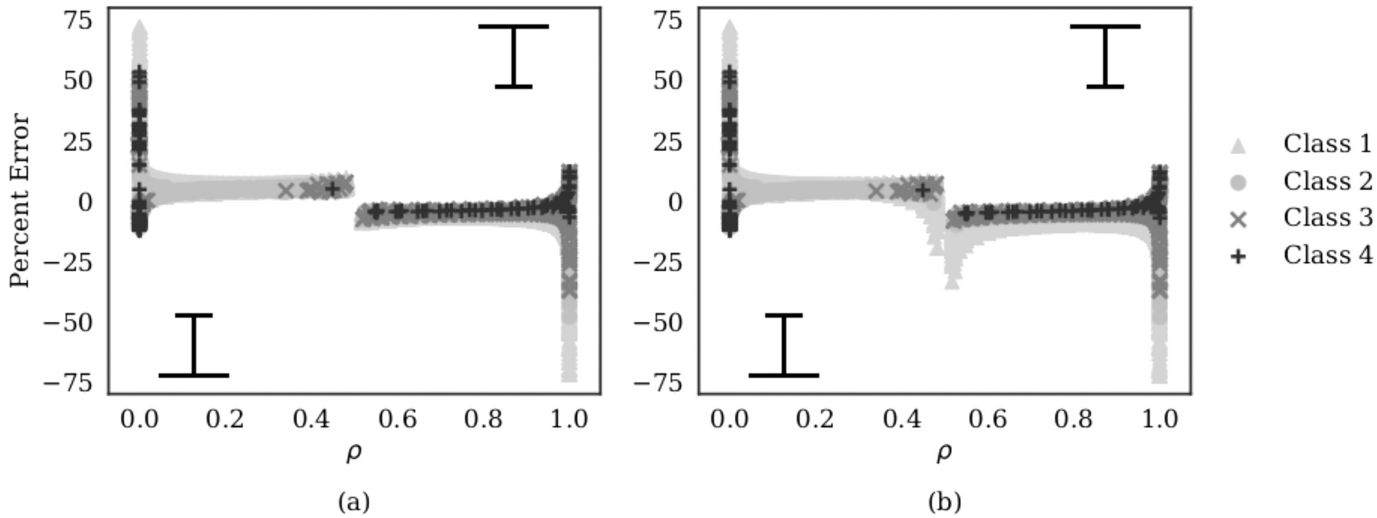
Fig. 2. Percent error of β_x for generated singly-symmetric I-sections: (*a*) eq. (6) (Kitipornchai and Trahair 1980) and (*b*) eq. (7) (CSA 2014*a*).



$$\beta_{x,K\&T} = 0.9(d - t)(2\rho - 1)\left(1 - \left(\frac{I_y}{I_x}\right)^2\right) \quad (6)$$

In deriving and calibrating eq. (6), the aforementioned researchers considered flange width-to-thickness ratios of $4 \leq \frac{b}{t} \leq 64$, flange-to-web-thickness ratios of $0.1 \leq \frac{t}{w} \leq 2.0$, and depth-to-web-thickness ratios of $10 \leq \frac{d}{w} \leq 290$. Of particular note, they excluded any beam cross-sections where $\frac{I_y}{I_x} > 0.5$ in the development of eq. (6). Within these constraints, a zero mean error with a standard deviation of 0.037 was reported for over 3000 beam cross-sections considered.

Standard S16 (CSA 2009) adopted eq. (6), albeit without any of the geometric restrictions imposed by the original researchers, and applied a further modification recommended by Kitipornchai and Trahair (1980) by replacing the parameter ρ by $\frac{I_{yc}}{I_y}$:

$$\beta_{x,S16} = 0.9(d - t)\left(2\frac{I_{yc}}{I_y} - 1\right)\left(1 - \left(\frac{I_y}{I_x}\right)^2\right) \quad (7)$$

Comparison study of asymmetry parameter, β_x

The accuracy of eqs. (6) and (7) was investigated by utilizing 281 W-shapes from the *Handbook of Steel Construction* (CISC 2017) to generate 16 312 singly-symmetric I-sections by independently reducing each flange in increments of 10 mm until each I-section reduces to a T-section. (For simplicity of analysis, fillets are neglected; the influence of this assumption is addressed subsequently.) These generated sections provide the starting point for the comparison study of β_x. The properties of the generated T-sections do not necessarily reflect the standard WT-shapes designated in the *Handbook* (CISC 2017), as their stems are generally much deeper; however, the method produces a broad set of reference geometries for singly-symmetric I-sections.

Generated singly-symmetric I-sections

Figure 2 shows that compared to the exact value eq. (3), eqs. (6) and (7) produce errors in the asymmetry parameter, β_x, that vary drastically depending on the degree of single-symmetry, ρ. The asymmetry of the curves generated from eq. (7) (Fig. 2*b*) reflects the fact that when $\rho = 0.5$, $\frac{I_{yc}}{I_y}$ is not equal to 0.5 due to the presence of the web, and therefore β_x is not equal to 0. As the value of β_x is small when ρ is near 0.5, the percent error can be large even if the absolute error is not; moreover, the sign of β_x generated from eq. (7) in this region of the graph can be opposite of its true sign. (Note that a negative percent error is consistent with a conservative beam moment capacity estimate for design, whereas a positive error is non-conservative in this respect. Arguably, however, when the sign of the β_x estimate becomes opposite of the true sign due to $\frac{I_y}{I_x} > 1.0$, it should be recognized by the designer as a gross inaccuracy and the concept of "conservatism" loses its common meaning.)

Of all singly-symmetric I-sections considered, if the yield stress is taken as 350 MPa, 1.8% are Class 4; nevertheless, no fundamental difference was observed from the outcomes for Classes 1, 2, and 3 sections. (It is noted that web classes are determined using the *Canadian Highway Bridge Design Code* (CSA 2014*b*), as these provisions are expected to be adopted into the 2019 edition of S16.) It is evident from Fig. 2, that both eqs. (6) and (7) can lead to large errors in β_x, particularly for T-sections (and for I-sections where one flange is very small).

Isolating the 562 generated T-sections from the full database, Fig. 3 shows that the percent errors are higher at larger $\frac{I_y}{I_x}$ ratios. As the generated T-sections tend to have very slender stems, 44% are Class 4 (99% of those have the stem in compression); however, since they follow similar trends to those of other section classes, they are retained in the analysis.

Standard WT-shapes

For standard WT-shapes, the asymmetry parameter, β_x, is provided explicitly in the *Handbook* (CISC 2017) section data tables and it includes the effect of the fillets. To assess the influence of the fillets on the value of β_x the *Handbook* values were compared to values determined using eq. (5), which neglects the fillets but is otherwise exact. The effect of the fillets on the value of β_x is less than 1% for all the WT-shapes and is thus considered negligible in this paper.

For standard WT-shapes potentially susceptible to LTB $\left(\frac{I_y}{I_x} < 1.0\right)$, the percent error ranges from +12.5% to −80% as shown in Fig. 4, where only cases with the flange in compression are included ($\rho = 1.0$). The large negative errors for large values of $\frac{I_y}{I_x}$ arise because the structure of the approximate equations is such that when $\frac{I_y}{I_x}$ approaches 1.0 the value of β_x approaches 0 and when $\frac{I_y}{I_x}$ exceeds 1.0 the value of β_x becomes negative even though

Fig. 3. Percent error of β_x for generated T-Sections: (*a*) eq. (6) (Kitipornchai and Trahair 1980) and (*b*) eq. (7) (CSA 2014*a*).

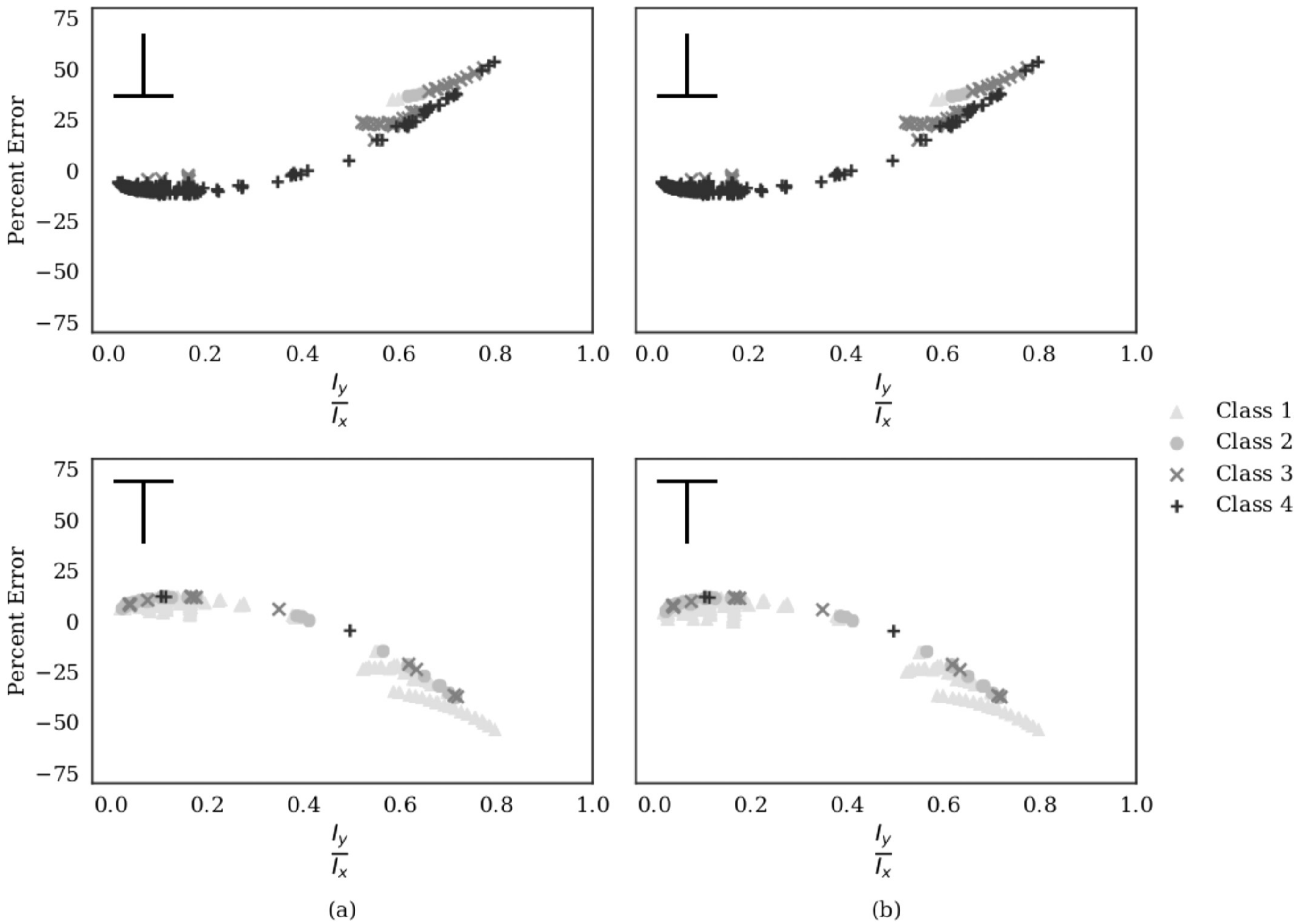
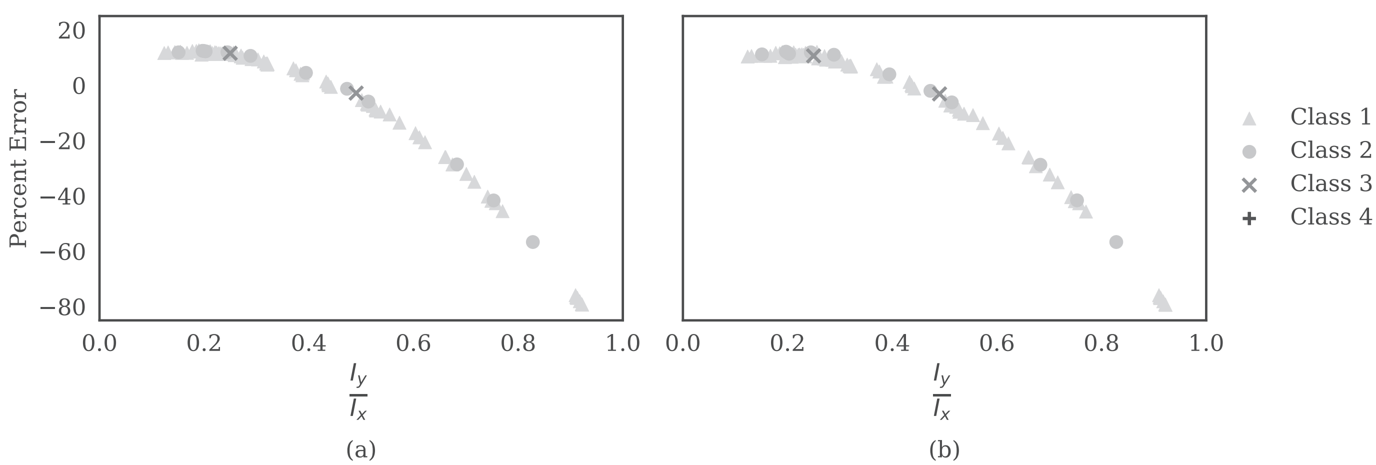
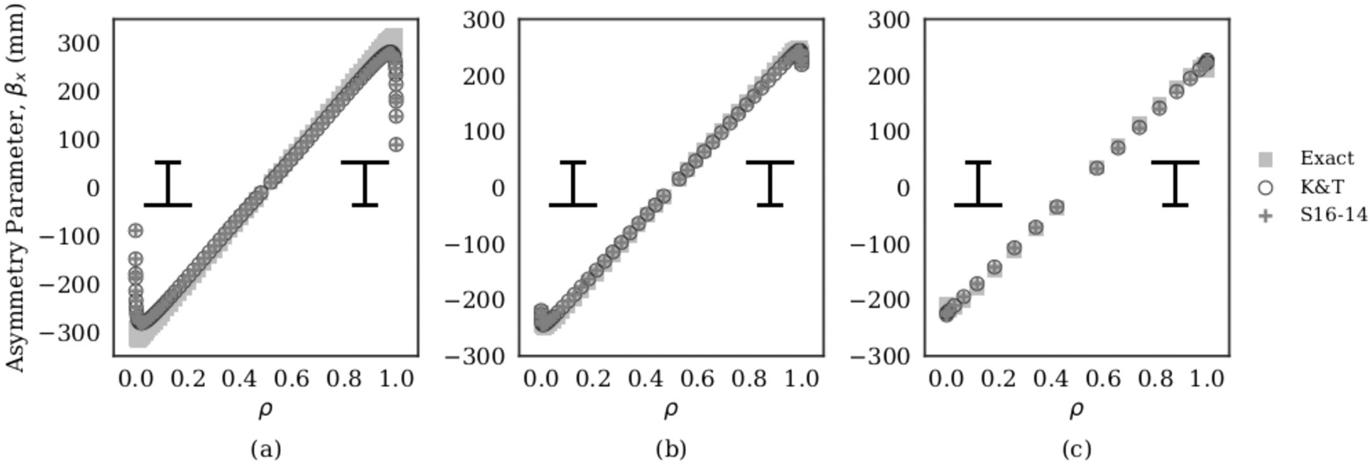


Fig. 4. Percent error of β_x for standard WT-shapes: (*a*) eq. (6) (Kitipornchai and Trahair 1980) and (*b*) eq. (7) (CSA 2014*a*).



the flange is in compression, which by definition corresponds to a positive β_x. As $\frac{I_y}{I_x}$ increases in magnitude beyond 1.0, both the magnitude and percent error of β_x increase rapidly with the error rising as high as –14 000% for $\frac{I_y}{I_x} > 5.0$, although this is only of theoretical interest since LTB would not occur. This observation reflects the fact that no cases where $\frac{I_y}{I_x} > 0.5$ were considered by Kitipornchai and Trahair (1980) in the development of eq. (6). Nevertheless, as indicated previously, negative errors are conservative for determining the design moment capacity of a beam. As indicated in Fig. 4, when $\frac{I_y}{I_x} < 0.5$ the negative errors do not

Fig. 5. β_x as a function of ρ for singly-symmetric sections generated from: (*a*) W1100×499, (*b*) W360×592, and (*c*) W250×25.



exceed 3.2%; however, positive errors (non-conservative) can reach 12.5%.

Discussion

From the foregoing analysis of singly-symmetric I-sections, including T-sections, it has been revealed that large errors can occur when determining the asymmetry parameter, β_x, by either eq. (6) (Kitipornchai and Trahair 1980) or eq. (7) (CSA 2014*a*). As shown in Fig. 2, the largest errors occur for T-sections—or I-sections with one flange very much smaller than the other—although errors are also large for sections that are nearly doubly-symmetric when determined using eq. (7). In the latter case, large yet conservative errors are of less concern when determining LTB capacity as the values of β_x themselves are small. Figure 3 shows that the greatest errors for T-sections are those corresponding to large values of $\frac{I_y}{I_x}$. The errors obtained using eqs. (6) and (7) are identical when $\rho = 0$, but are slightly different when $\rho = 1.0$. Figure 4 indicates that when using standard WT-shapes with the flange in compression, very large errors occur using both eqs. (6) and (7). Although the largest of these errors occur when $\frac{I_y}{I_x}$ is large, the greatest positive error (non-conservative) occurs for small values of $\frac{I_y}{I_x}$.

When evaluating the accuracy of the approximate equations for estimating the asymmetry parameter, it must be borne in mind that errors in the value of β_x do not translate directly to errors in the calculated design moment capacity. As a preliminary evaluation of the effect on the elastic critical buckling moment of using eqs. (6) and (7) to estimate β_x, all cross-sections considered were analyzed under a constant moment using the shortest beam length for which elastic LTB (neglecting residual stresses) is the governing limit state. Errors in the critical moments were found to be within the range of +7.5% (non-conservative) to −10% (conservative), with extreme singly-symmetric sections (e.g., T-sections) showing the greatest error. Other aspects that may further mitigate the impact of errors in the estimation of β_x include inelastic LTB behaviour and moment gradient effects. Also, the common case of orienting the beam so that the larger flange is in compression results in a conservative estimate of β_x, with the exception of T-sections with small values of $\frac{I_y}{I_x}$.

By plotting β_x values of three standard W-shapes with substantially different geometric properties and their corresponding generated singly-symmetric sections, the characteristic shape of the β_x versus ρ curve becomes evident, as seen in Fig. 5. While the curve is relatively linear over most of the range of ρ, for extreme singly-symmetric I-sections and T-sections this relationship tends to break down.

Mathematically, the large percent errors generated by eqs. (6) and (7) for extreme singly-symmetric I-sections are exacerbated by the extreme non-linearity of β_x near the extremities of the interval $0 \le \rho \le 1.0$. This raises the possibility of limiting the application of either eq. (6) or (7) to within certain limits based on the value of ρ. Figure 2 shows that limits of $0.1 \le \rho \le 0.9$ would exclude the most extreme errors in the values of β_x on both the conservative and non-conservative sides. If the equations are then further restricted only to cases where the larger flange is in compression ($0.5 < \rho \le 0.9$), all non-conservative cases are eliminated.

Conclusion

The equation in the current Canadian design standard S16 (CSA 2014*a*) for estimating the asymmetry parameter, β_x, for singly-symmetric sections subjected to flexure has been observed to provide significant errors, especially for T-sections. However, for I-sections with the larger flange in compression, the error generally results in a conservative estimate of the moment capacity of the beam for use in design.

While exact methods are available for determining β_x, they tend to be cumbersome in a design context. To enforce the benchmarks of $\beta_x = 0$ for doubly-symmetric beams and $\rho = 1.0$ for T-sections with the flange in compression, it is recommended that in the 2019 edition of CSA Standard S16 eq. (6) be adopted as an alternative to more accurate methods with the added restriction that $0.1 < \rho \le 0.9$. Further restricting this range to $0.5 < \rho \le 0.9$ would limit errors in β_x to those that are conservative for design, but even over the full proposed range the error does not exceed 9%. For WT-shapes, accurate values of β_x are tabulated in the *Handbook* (CISC 2017) and for custom T-sections eq. (5) can be used. Further research is needed to provide a complete understanding of the effect of using approximate values of β_x on the design LTB capacity of beams with singly-symmetric I-sections, and T-sections.

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